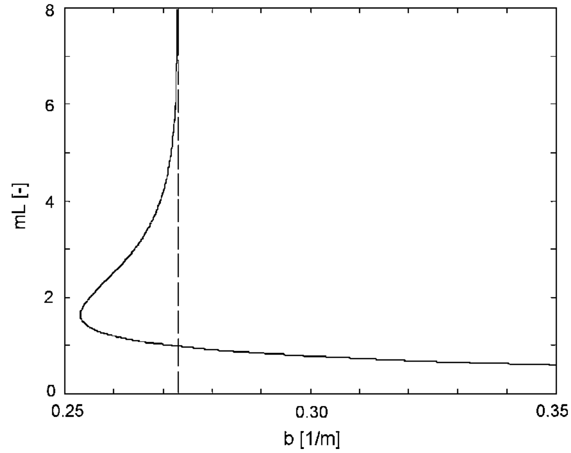

Fig. 2 Values of the total dimensionless fin length *mL* satisfying all of the boundary conditions, as a function of the parameter *b*, proportional to the heat flux at the fin root.

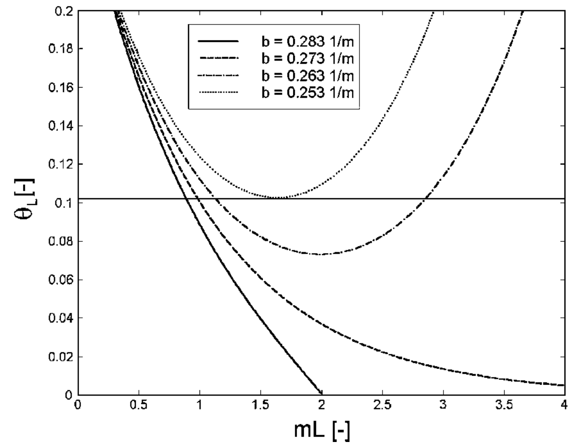

Fig. 3 Admissible values of the dimensionless temperature at the tip as a function of the fin length for different values of the parameter *b*; the solution of the problem is given by the intersection of the curves with the horizontal line at $\theta_L = 0.102$.

Conclusions

Although the heat conduction problem in cooling fins is usually considered to have one steady-state solution for given boundary conditions, two solutions may exist when nonstandard, yet physically admissible, boundary conditions are applied. In particular, the situation in which the cooling fin has fixed temperature at both ends and specified heat flux at the root is examined. The analysis shows that, depending on the value of the heat flux at the fin root, the problem may have no solutions, one solution, or two solutions for the fin length. The results can find practical applications in the design of passive safety devices for chemical or nuclear plants.

Acknowledgments

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Integral Equation for the Heat Transfer with the Moving Boundary

Vladimir V. Kulish*
Nanyang Technological University, Singapore 639798, Republic of Singapore
and
Vasily B. Novozhilov†
University of Ulster, Newtownabbey, England BT37 0QB, United Kingdom

Nomenclature

A	=	heated surface area, m^2
b	=	mean of normal distribution, s
c_p	=	specific heat of solid material
E	=	activation energy, J
F	=	fluence of the laser, J
K	=	preexponent
k	=	thermal conductivity, W m^{-1} K^{-1}
p	=	pressure, Pa
q''	=	local heat flux, W m^{-2}
q_s''	=	surface heat flux (at $x = 0$), W m^{-2}
R	=	gas constant
r	=	reflectivity of the material
T	=	absolute temperature, K
T_{max}	=	maximum temperature during heating, K
T_s	=	surface temperature (at $x = 0$), K
T_0	=	initial temperature (at $t \le 0$), K
t	=	time, s
u	=	speed of the domain boundary, m s^{-1}
x	=	coordinate normal to material surface, m
α	=	thermal diffusivity, m^2 s^{-1}
θ	=	excess temperature, $= T - T_0$, K
ϑ	=	normalized nondimensional temperature, $= (T_s - T_0)/(T_m - T_0)$
μ, ν	=	given parameters for a particular fuel
ρ	=	density, kg m^{-3}
σ	=	variance of normal distribution, s

Introduction

PROBLEMS of heat transfer in domains with moving boundaries are commonly found in scientific and engineering applications. As typical examples, one can consider the following classes of problems:

1) Stefan-type and related problems. These typically arise when there is a phase change at the boundaries between media with different conducting properties, for example, during melting/solidification of alloys or warming/freezing of water-containing soils.

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*Associate Professor, School of Mechanical and Production Engineering, 50 Nanyang Avenue; mvvkulish@ntu.edu.sg or kulishv@asme.org.

†Professor, Faculty of Engineering and Built Environment, Jordanstown Campus, Shore Road.

2) Evaporation of liquid layers and droplets under external heat flux conditions.

3) Combustion of solids and liquids, which cause fuel surface regression. This phenomenon occurs, for example, during pool and tank fires, as well as for burning plastics and solid propellants.

A convenient and effective way of solving such problems is reduction to the sets of integral or integro–differential equations, containing unknown properties at boundary locations. The primary importance of such methods is due to the removal of unnecessary numerical complications (such as adjustment of the grid to moving boundaries, grid refinement in the regions of sharp gradients, necessity to make grid refinement tests, etc.).

In the present Note, a relationship between the local temperature and the heat flux, obtained earlier for a semi-infinite conducting solid,[1] is expanded to the case of the domain with a moving boundary. The obtained relationship leads to an integral equation at the moving boundary, which can be used for convenient calculation of the surface temperature.

Solution of the Problem

Consider a one-dimensional semi-infinite domain whose boundary is moving with a constant velocity u. This domain is initially, at time $t = 0$, in thermal equilibrium with the temperature $T = T_0$. The surface heating of the domain starts at time $t = 0$, and the value of heat flux is $q_s''(t)$.

In this case, the energy equation assumes the form

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial x^2} \tag{1}$$

where α is the domain thermal diffusivity.

After introducing the excess temperature, defined as $\theta = T - T_0$, and a new "spatial" variable, $\xi = x/\sqrt{\alpha}$, the solution to Eq. (1) can be found by using the Laplace transform technique (see Ref. 1 for details). The solution is in the form of an integral equation that relates temperature and heat flux, namely,

$$T(x,t) = T_0 + \int_0^t q''(x,\tau)\left[\frac{e^{(-u^2/4\alpha)(t-\tau)}}{\sqrt{\pi k\rho c_p(t-\tau)}} + \frac{u}{2k}\mathrm{erfc}\left(-\frac{u}{2}\sqrt{\frac{t-\tau}{\alpha}}\right)\right]\mathrm{d}\tau \tag{2}$$

Equation (2) provides one with the relationship between the local values of temperature and heat flux and is true everywhere in a one-dimensional semi-infinite domain whose boundary is moving with a constant velocity. It is noteworthy to emphasize here that the relationship given by Eq. (2) is valid everywhere, including the domain boundary. Thus, the surface temperature $T_s(t)$ of the domain under consideration can be found for a given surface heat flux $q_s''(t)$ as

$$T_s(t) = T_0 + \int_0^t q_s''(\tau)\left[\frac{e^{(-u^2/4\alpha)(t-\tau)}}{\sqrt{\pi k\rho c_p(t-\tau)}} + \frac{u}{2k}\mathrm{erfc}\left(-\frac{u}{2}\sqrt{\frac{t-\tau}{\alpha}}\right)\right]\mathrm{d}\tau \tag{3}$$

Observe also from Eq. (2) that if the domain boundary is at rest, that is, $u = 0$, then the solution becomes

$$T(x,t) = T_0 + \int_0^t \frac{q''(x,\tau)}{\sqrt{\pi k\rho c_p(t-\tau)}}\mathrm{d}\tau \tag{4a}$$

which, on the other hand, can be written in terms of fractional (of noninteger order) derivatives as

$$T_s(t) = T_0 + \frac{1}{\sqrt{k\rho c_p}}\frac{\partial^{-\frac{1}{2}}q''(x,t)}{\partial t^{-\frac{1}{2}}} \tag{4b}$$

the result that was reported in Ref. 1.

Physical Interpretation

As one example of physical interpretation of the solutions (2) and (3), consider combustion of a uniform solid propellant, described by Eq. (1). It is well known[2] that the surface temperature of the propellant may vary with time, and consideration of this effect is quite important in real applications.

The linear regression rate of the propellant's surface, u, depends on a number of external factors, such as pressure in the combustion chamber, as well as on the properties of the fuel itself. Different correlations have been proposed to describe burning rates of solid propellants. Typical models[2] assume linear burning rate in the form

$$u = K\rho^{-1}p^{\nu}T_s^{\mu}\exp[-E/(2RT_s)] \tag{5}$$

In this relationship, both surface temperature and pressure are generally functions of time.

It is apparent from Eq. (5) that the constant burning rate u occurs when the gas pressure in the combustion chamber varies with time as

$$p(t) = \left(\mathrm{const}\cdot K^{-1}\rho[T_s(t)]^{-\mu}\exp\{E/[2RT_s(t)]\}\right)^{1/\nu} \tag{6}$$

One can also consider this example as a combustion control problem. For stability of combustion and good rocket engine performance, it is usually desirable to achieve steady-state regimes of burning. Under such a set of the problem, the external heat flux $q_s''(t)$ and the chamber pressure $p(t)$ can be viewed as controlling parameters, which can be varied with time in order to achieve the required rate of burning. The heat flux and pressure would control fuel conditions, such as surface temperature and burning rate. The required control function $[q_s''(t),\ p(t)]$ can be found using Eqs. (3) and (6).

Numerical Simulation

The solution for the surface temperature, given by Eq. (3), was tested by numerically solving several cases implying different sets of boundary data for the heat flux.

To be consistent with the results obtained previously, the physical properties of a GaAs bulk sample ($r = 0.39$, $\rho c_p = 1.73\ 10^6$ J/m^{3}K, $k = 52$ W m^{-1} K^{-1}) were used in the course of the numerical simulation (see Ref. 3 for details).

First, the surface temperature was computed in the case of a constant heat flux value, $q_s'' = 10^3$ W m^{-2}, for three different values of the boundary speed u: 0, 1.0, and 10.0 m s^{-1}, respectively. The results are shown in Fig. 1.

Although at the same time the absolute values of the surface temperature are different for different values of the boundary speed, the surface temperature behavior follows $T_s(t) \sim \sqrt{t}$, which indeed must be the case for a constant value of the surface heat flux. Moreover, the result, in the case of zero boundary speed, matches the result reported in Ref. 1 for the same case [see Eq. (4)].

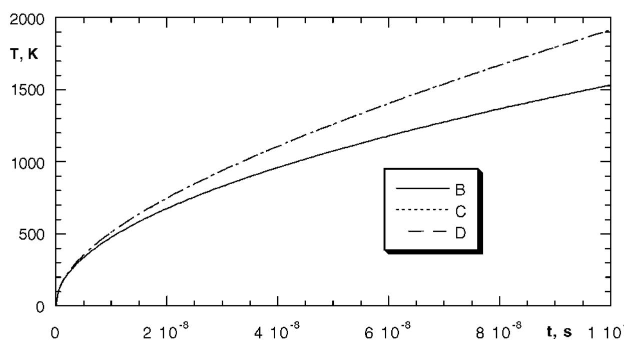

Fig. 1 Time evolution of the surface temperature in the case of a constant heat flux: B, zero boundary speed; C, boundary speed 1.0 m s^{-1}; and D, boundary speed 10.0 m s^{-1}.

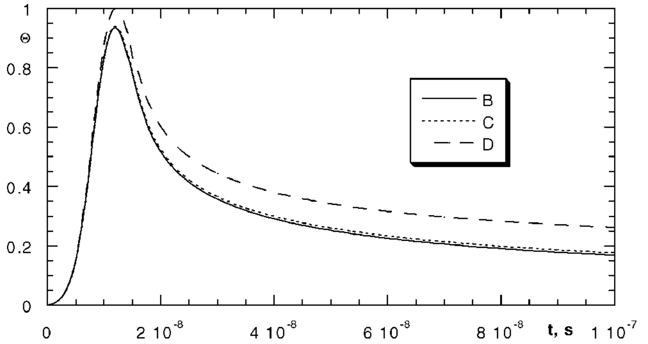

Fig. 2 Time evolution of the surface temperature in the case of a pulsing heat flux: B, zero boundary speed; C, boundary speed 1.0 m s^{-1}; and D, boundary speed 10.0 m s^{-1}.

The next case, computed numerically, was a simulation of the fast laser heating of the surface. The surface heat flux was given by the Gaussian distribution, in exactly the same manner as it was done in Ref. 3, that is,

$$q_s''(t) = \frac{F(1-r)}{2A\sigma\sqrt{\pi}} \exp\left[-\left(\frac{t-b}{\sigma}\right)^2\right] \tag{7}$$

This situation mimics a pulsed Nd:YAG laser radiating at 532-nm wavelength used as a heating source, with a fluence F that can be set from 0 to 1.0 mJ. The parameters of the laser pulse were chosen in order to be consistent with the results reported in Ref. 3.

The numerical results, representing the time evolution of the dimensionless excess surface temperature, $\theta = (T - T_0)/(T_{\max} - T_0)$, are shown in Fig. 2 for three different values of the boundary speed u:0, 1.0, and 10.0 m s^{-1}.

The solution represented by curve B (zero speed of the boundary) coincides with the solution reported in Ref. 3 and given by Eq. (4).

Conclusions

Using the same method that was initially proposed in Ref. 1, the relationship between the local temperature and local heat flux has been established for the homogeneous one-dimensional heat equation in a semi-infinite domain whose boundary moves with a constant velocity. This relationship has been written in the form of a convolution integral.

The integral equation, relating the surface temperature and surface heat flux, has been solved numerically in the case of a constant heat flux value and in the case of a pulsing heat flux, mimicking fast laser heating of solid materials.

The results obtained in the course of the numerical simulation show the following:

1) These results coincide with those results obtained by the same method in the case of a nonmoving boundary.

2) The value of the boundary speed influences the surface temperature value—if the former increases, the latter increases too—but not the shape of the time evolution curve.

3) The moving boundary effect is not very significant in the case of fast laser heating due to the shortness of the process.

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Thermal Conductance Across Oxygen-Free High-Conductivity Copper Contacts in Different Environments

Bapurao Kshirsagar,* J. Nagaraju,† and M. V. Krishna Murthy‡

Indian Institute of Science, Banglaore 560 012, India

Nomenclature

A_c	=	apparent contact area
H_c	=	microhardness of contacting surface
h_c	=	thermal contact conductance
h_g	=	gap conductance
k	=	thermal conductivity
m	=	average asperity slope
P	=	contact pressure
Q	=	heat flow rate
σ	=	rms value of surface roughness

Introduction

WHEN two metallic surfaces are brought into contact to transmit heat, the uniform flow of heat is restricted to conduction through the contact spots, which are limited in number and size. The magnitude of contact conductance is a function of a number of parameters including the thermophysical and mechanical properties of the materials in contact, the characteristics of the contacting surfaces, the presence of interstitial media, the contact pressure, the mean interface temperature, and the conditions surrounding the contact. Fletcher[1] reviewed the various experimental techniques to measure the thermal contact resistance. Mikic[2] investigated theoretically the effect of mode of deformation on the predicted values of thermal contact conductance and suggested a correlation for both plastic and elastic deformations. Sridhar and Yovanovich[3] proposed an elastoplastic contact–conductance model for isotropic and conforming rough surfaces. Wahid and Madhusudana[4] gave a relation between the effective gap thickness and surface roughness for all gases. In view of the significant number of parameters affecting the thermal contact conductance, it is mostly determined experimentally to provide a measure of the thermal performance of a specific configuration. The objective of the present investigation is to study the variation in thermal conductance across oxygen-free high-conductivity (OFHC) copper contacts in vacuum, nitrogen, and helium environments.

Experimentation

A schematic of the thermal contact conductance measurement test setup is shown in Fig 1. It consists of a contact conductance cell, a hydraulic loading unit, a heating circuit, a cooling circuit, a vacuum system, and instrumentation. The test column assembly consists of a pair of heater–cooler blocks, test specimens, heat-flux meters made of OFHC copper, and a pair of ball-cone seat arrangement on either side. The heater–cooler block can be used either as a heater or as a cooler. A triple-walled chamber made of AISI304 accommodates

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*Research Student, Department of Instrumentation.

†Assistant Professor, Department of Instrumentation.

‡Professor, Department of Instrumentation.